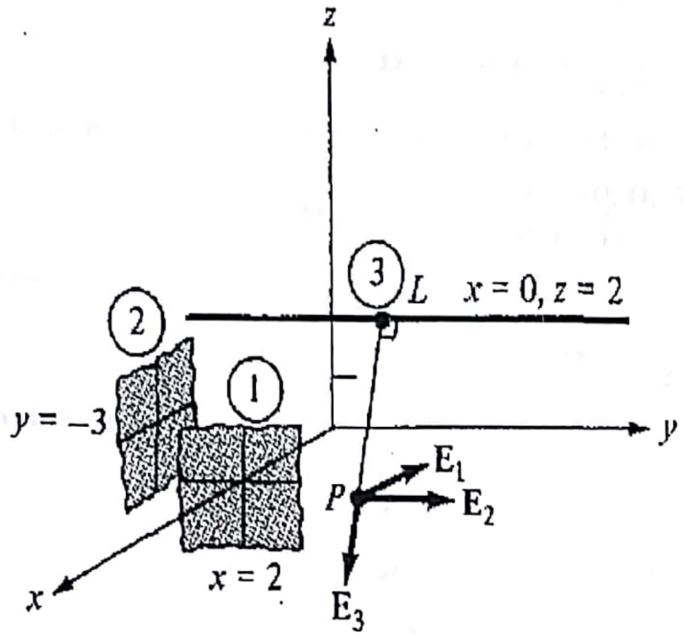
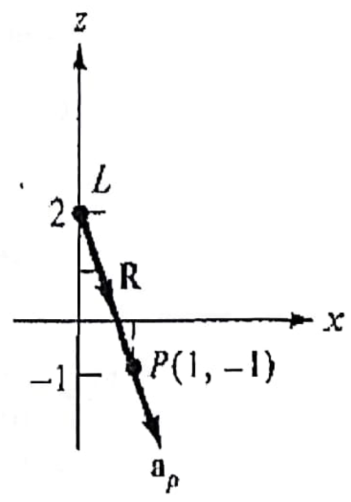
اسماعيل كرار مصطفى

Solution

1- Planes x=2 and y=-3, respectively, carry charges 10 nC/m^2 and 15 nC/m^2. If the line x=0, z=2 carries charge $10\pi\ nC/m$, and a point charge $2\ nC$ at P(0,0,0), calculate $\boldsymbol{E}$ at (1,1,-1).

$E=E_1+E_2+E_3+E_4$





$$\mathbf{E}_1 = \frac{\rho_{S_1}}{2\varepsilon_o}(-\mathbf{a}_x) = -\frac{10\cdot 10^{-9}}{2\cdot\frac{10^{-9}}{36\pi}}\mathbf{a}_x = -180\pi\mathbf{a}_x$$

$$\mathbf{E}_2 = \frac{\rho_{S_2}}{2\varepsilon_o}\mathbf{a}_y = \frac{15\cdot 10^{-9}}{2\cdot\frac{10^{-9}}{36\pi}}\mathbf{a}_y = 270\pi\,\mathbf{a}_y$$

$$\mathbf{R} = -3\mathbf{a}_z + \mathbf{a}_x$$

$$\rho = |\mathbf{R}| = \sqrt{10}, \qquad \mathbf{a}_\rho = \frac{\mathbf{R}}{|\mathbf{R}|} = \frac{1}{\sqrt{10}}\mathbf{a}_x - \frac{3}{\sqrt{10}}\mathbf{a}_z$$

$$\mathbf{E}_3 = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho$$

$$\mathbf{E}_3 = \frac{10\pi\cdot 10^{-9}}{2\pi\cdot\frac{10^{-9}}{36\pi}}\cdot\frac{1}{10}(\mathbf{a}_x - 3\mathbf{a}_z) = 18\pi(\mathbf{a}_x - 3\mathbf{a}_z)$$

$$E_4 = \frac{Q}{4\pi\varepsilon r^2}a_r = \frac{6(a_x+a_y-a_z)}{\sqrt{3}}$$

Total E(1,1,-1) =-505.474 a_x +851.694 a_y – 173.11a_z V/m

2- Determine **D** at (4, 0, 3) if there is a point charge -5 π mC at (4, 0, 0) and a line charge 3 π mC/m along the y-axis.

Solution:

Let $\mathbf{D} = \mathbf{D_Q} + \mathbf{D_L}$ where $\mathbf{D_Q}$ and $\mathbf{D_L}$ are flux densities due to the point charge and line charge, respectively, as shown in Figure:

$$\mathbf{D}_Q = \varepsilon_o \mathbf{E} = \frac{Q}{4\pi R^2}\mathbf{a}_R = \frac{Q\,(\mathbf{r} - \mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|^3}$$

where $\mathbf{r} - \mathbf{r}' = (4, 0, 3) - (4, 0, 0) = (0, 0, 3)$. Hence,

$$\mathbf{D}_Q = \frac{-5\pi \cdot 10^{-3}(0, 0, 3)}{4\pi|(0, 0, 3)|^3} = -0.138\ \mathbf{a}_z\ \text{mC/m}^2$$

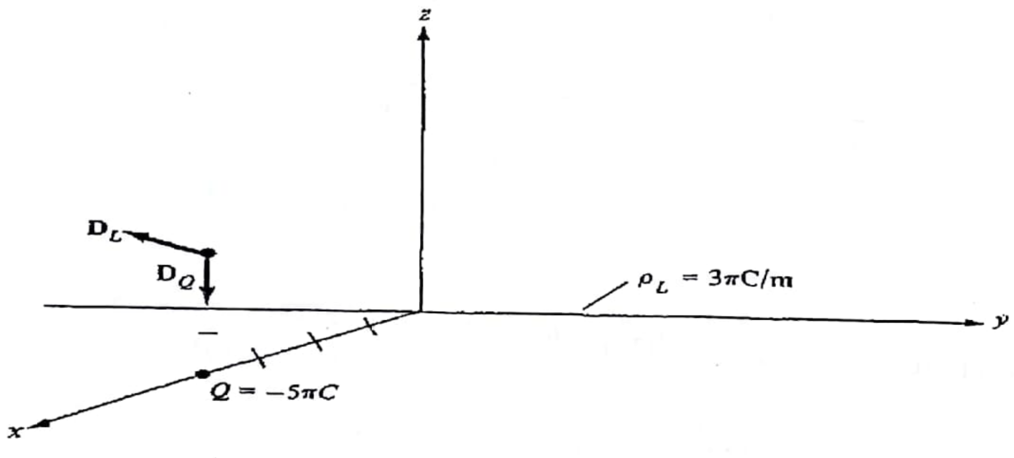
Also

$$\mathbf{D}_L = \frac{\rho_L}{2\pi\rho}\mathbf{a}_\rho$$

In this case

$$\mathbf{a}_\rho = \frac{(4, 0, 3) - (0, 0, 0)}{|(4, 0, 3) - (0, 0, 0)|} = \frac{(4, 0, 3)}{5}$$

$$\rho = |(4, 0, 3) - (0, 0, 0)| = 5$$



Hence,

$$\mathbf{D}_L = \frac{3\pi}{2\pi(25)}(4\mathbf{a}_x + 3\mathbf{a}_z) = 0.24\mathbf{a}_x + 0.18\mathbf{a}_z\ \text{mC/m}^2$$

Thus

$$\mathbf{D} = \mathbf{D}_Q + \mathbf{D}_L$$
$$= 240\mathbf{a}_x + 42\mathbf{a}_z\ \mu\text{C/m}^2$$

3-

a) the equation of the streamline passing through the point $A(2, 1, -2)$: Write:

$$\frac{dy}{dx} = \frac{E_y}{E_x} = \frac{x}{y} \Rightarrow x\,dx = y\,dy$$

Thus $x^2 = y^2 + C$. Evaluating at A yields $C = 3$, so the equation becomes

$$\frac{x^2}{3} - \frac{y^2}{3} = 1$$

A sketch of the part *a* equation would yield a parabola, centered at the origin, whose axis is the positive x axis, and for which the slopes of the asymptotes are ± 1.

4-

(a) $\mathbf{r}_P = 0\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z = 2\mathbf{a}_y + 4\mathbf{a}_z$

(b) $\mathbf{r}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P = (-3, 1, 5) - (0, 2, 4) = (-3, -1, 1)$
or $\mathbf{r}_{PQ} = -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$

(c) Since $\mathbf{r}_{PQ}$ is the distance vector from P to Q, the distance between P and Q is the magnitude of this vector; that is,

$$d = |\mathbf{r}_{PQ}| = \sqrt{9 + 1 + 1} = 3.317$$

Alternatively:

$$d = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2}$$
$$= \sqrt{9 + 1 + 1} = 3.317$$

(d) Let the required vector be $\mathbf{A}$, then

$$\mathbf{A} = A\mathbf{a}_A$$

where $A = 10$ is the magnitude of $\mathbf{A}$. Since $\mathbf{A}$ is parallel to PQ, it must have the same unit vector as $\mathbf{r}_{PQ}$ or $\mathbf{r}_{QP}$. Hence,

$$\mathbf{a}_A = \pm\frac{\mathbf{r}_{PQ}}{|\mathbf{r}_{PQ}|} = \pm\frac{(-3, -1, 1)}{3.317}$$

and

$$\mathbf{A} = \pm\frac{10(-3, -1, -1)}{3.317} = \pm(-9.045\mathbf{a}_x - 3.015\mathbf{a}_y + 3.015\mathbf{a}_z)$$

5- Determine the gradient of the following scalar field $V = \rho z \sin\varphi + z^2\cos^2\varphi + \rho^2$

(b) $(z\sin\phi + 2\rho)\mathbf{a}_\rho + \left(z\cos\phi - \frac{z^2}{\rho}\sin 2\phi\right)\mathbf{a}_\phi + (\rho\sin\phi + 2z\cos^2\phi)\mathbf{a}_z$