Ministry of Higher Education
The Higher Institute of Engineering and Technology in New Damietta
Course title: Mechanics-1
Course code: ENG.101
Semester: Summer Midterm

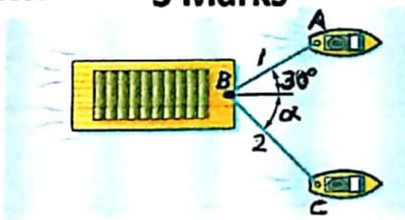
Department: General
Level: 1
Time allowed: 90 Minutes
Date: 28/7/2018 Day: Sat.
Full Mark: 20
No. of exam pages: 1

Q:1 .... 5 Marks

A barge is pulled by two tugboats. If the resultant of the forces exerted by tugboats is a 5000 N directed along the axis of the barge, **determine:**

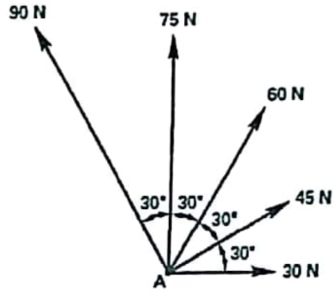
A) The tension in each of the ropes knowing that $\alpha = 45°$,

B) The value of α for which the tension in rope 2 is minimum.



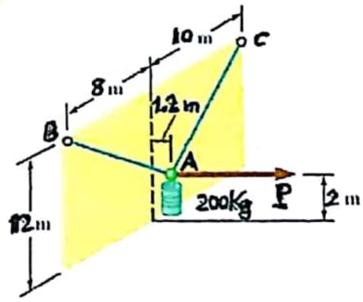
Q:2 .... 5 Marks

The five forces shown act at point A. **What** is the magnitude of the resultant force, and **what** is its angle to the x-axis.



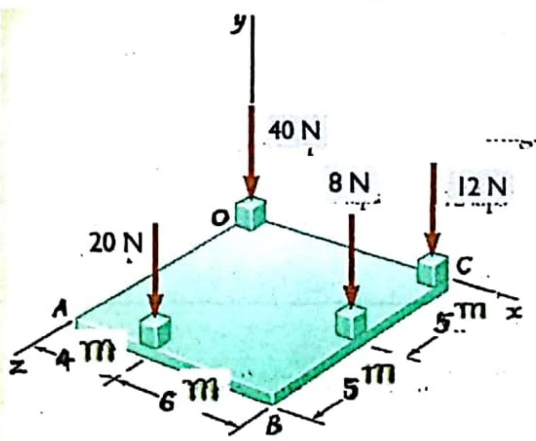
Q:3 .... 5 Marks

A 200 kg cylinder is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force P perpendicular to the wall holds the cylinder in the position shown. **Determine** the magnitude of P and the tension in each cable.



Q:4 .... 5 Marks

A square foundation mat supports the four columns shown. Determine the magnitude and point of application of the resultant of the four loads.

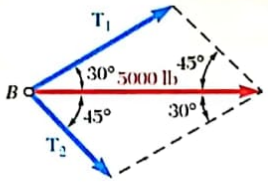


Best wishes,

Dr. Salah Dafea

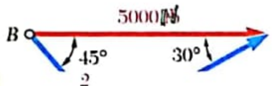
Model Answer of Mechanics-1 (ENG.101)

ANSWER Q:1 **5 Marks**



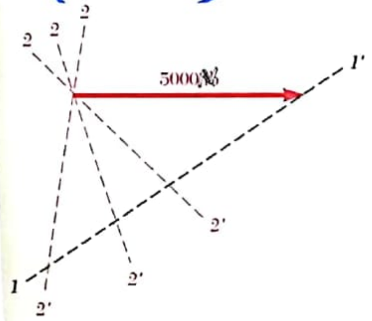
a. Tension for $\alpha = 45°$. *Graphical Solution.* The parallelogram law is used; the diagonal (resultant) is known to be equal to 5000 N and to be directed to the right. The sides are drawn parallel to the ropes. If the drawing is done to scale, we measure

$$T_1 = 3660 \text{ N} \qquad T_2 = 2590 \text{ N}$$



Trigonometric Solution. The triangle rule can be used. We note that the triangle shown represents half of the parallelogram shown above. Using the law of sines, we write

T_1 $\qquad$ T_2 $\qquad$ 5000 N



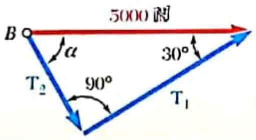
b. Value of α for Minimum T_2. To determine the value of α for which the tension in rope 2 is minimum, the triangle rule is again used. In the sketch shown, line *1-1'* is the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are shown by the lines *2-2'*. We note that the minimum value of T_2 occurs when $\mathbf{T}_1$ and $\mathbf{T}_2$ are perpendicular. The minimum value of T_2 is

$$T_2 = (5000 \text{ N}) \sin 30° = 2500 \text{ N}$$

Corresponding values of T_1 and α are

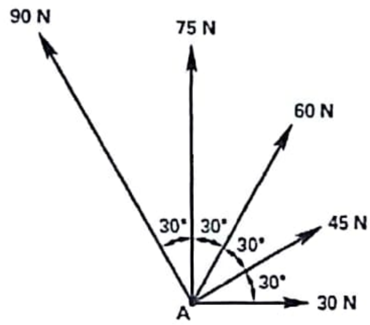
$$T_1 = (5000 \text{ N}) \cos 30° = 4330 \text{ N}$$

$$\alpha = 90° - 30° \qquad \alpha = 60°$$



<u>***ANSWER Q:2***</u>

The five forces shown act at point A. What is the magnitude of the resultant force?



Solution

$$\sum F_x = 30\ \text{N} + (45\ \text{N})\cos 30^\circ + (60\ \text{N})\cos 60^\circ + (75\ \text{N})\cos 90^\circ + (90\ \text{N})\cos 120^\circ$$

$$= 54\ \text{N}$$

$$\sum F_y = (30\ \text{N})\sin 0^\circ + (45\ \text{N})\sin 30^\circ + (60\ \text{N})\sin 60^\circ + 75\ \text{N} + (90\ \text{N})\sin 120^\circ$$

$$= 227.4\ \text{N}$$

$$R = \sqrt{(54\ \text{N})^2 + (227.4\ \text{N})^2}$$

$$= 233.7\ \text{N} \quad (234\ \text{N})$$

$$\cos\theta = \frac{54}{233.7}$$

$$\theta = \cos^{-1}\left(\frac{54}{233.7}\right)$$

SOL Q:3

Free-body Diagram. Point A is chosen as a free body; this point is subjected to four forces, three of which are of unknown magnitude.

Introducing the unit vectors **i**, **j**, **k**, we resolve each force into rectangular components.

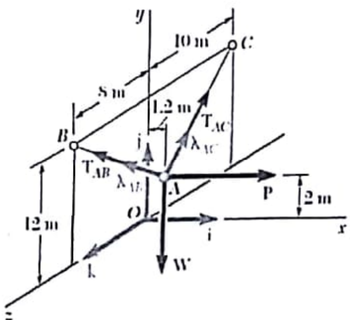
$$\mathbf{P} = P\mathbf{i} \tag{1}$$
$$\mathbf{W} = -mg\mathbf{j} = -(200\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(1962\text{ N})\mathbf{j}$$

In the case of $\mathbf{T}_{AB}$ and $\mathbf{T}_{AC}$, it is necessary first to determine the components and magnitudes of the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Denoting by $\boldsymbol{\lambda}_{AB}$ the unit vector along AB, we write

$$\overrightarrow{AB} = -(1.2\text{ m})\mathbf{i} + (10\text{ m})\mathbf{j} + (8\text{ m})\mathbf{k} \qquad AB = 12.862\text{ m}$$
$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{12.862\text{ m}} = -0.09330\mathbf{i} + 0.7775\mathbf{j} + 0.6220\mathbf{k}$$
$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = -0.09330T_{AB}\mathbf{i} + 0.7775T_{AB}\mathbf{j} + 0.6220T_{AB}\mathbf{k} \tag{2}$$

Denoting by $\boldsymbol{\lambda}_{AC}$ the unit vector along AC, we write in a similar way

$$\overrightarrow{AC} = -(1.2\text{ m})\mathbf{i} + (10\text{ m})\mathbf{j} - (10\text{ m})\mathbf{k} \qquad AC = 14.193\text{ m}$$
$$\boldsymbol{\lambda}_{AC} = \frac{\overrightarrow{AC}}{14.193\text{ m}} = -0.08455\mathbf{i} + 0.7046\mathbf{j} - 0.7046\mathbf{k}$$
$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = -0.08455T_{AC}\mathbf{i} + 0.7046T_{AC}\mathbf{j} - 0.7046T_{AC}\mathbf{k} \tag{3}$$



Equilibrium Condition. Since A is in equilibrium, we must have

$$\Sigma\mathbf{F} = 0: \qquad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$$

or, substituting from (1), (2), (3) for the forces and factoring **i**, **j**, **k**,

$$(-0.09330T_{AB} - 0.08455T_{AC} + P)\mathbf{i} + (0.7775T_{AB} + 0.7046T_{AC} - 1962\text{ N})\mathbf{j} + (0.6220T_{AB} - 0.7046T_{AC})\mathbf{k} = 0$$

$$(\Sigma F_x = 0:) \qquad -0.09330T_{AB} - 0.08455T_{AC} + P = 0$$
$$(\Sigma F_y = 0:) \qquad +0.7775T_{AB} + 0.7046T_{AC} - 1962\text{ N} = 0$$
$$(\Sigma F_z = 0:) \qquad +0.6220T_{AB} - 0.7046T_{AC} = 0$$

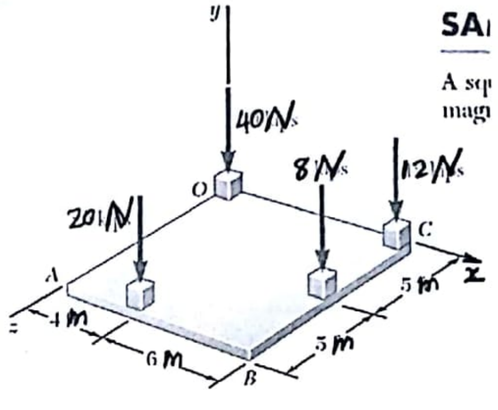
Solving these equations, we obtain

$$P = 235\text{ N} \qquad T_{AB} = 1402\text{ N} \qquad T_{AC} = 1238\text{ N}$$

SOL Q:4

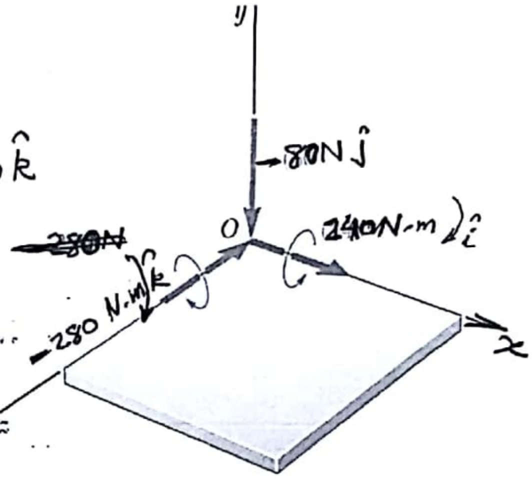
$R = \Sigma F$

$M)_O = \Sigma (r \times F)$



SA
A sq
mag

$\vec{r}$ ($\hat{i}$ $\hat{j}$ $\hat{k}$)	$\vec{F}$ ($\hat{i}$ $\hat{j}$ $\hat{k}$)	$r \times \vec{F}$
$0\hat{i}$	$-40\hat{j}$	0
$10\hat{i}$	$-12\hat{j}$	$-120\hat{k}$
$10\hat{i} + 5\hat{k}$	$-8\hat{j}$	$40\hat{i} - 80\hat{k}$
$4\hat{i} + 10\hat{k}$	$-20\hat{j}$	$200\hat{i} - 80\hat{k}$
	$\Sigma F = -80\hat{j}$	$\Sigma M)_O = 240\hat{i} - 280\hat{k}$



Dealing with $\vec{R}$:

$r \times \vec{R} = M_R)$

$(x\hat{i} + z\hat{k}) \times (-80\hat{j}) = 240\hat{i} - 280\hat{k}$

from which : $-80x = -280$

$x = 3.5$ m

$80z = 240$

$z = 3$ m

$R = 80$ N ↓ at $x = 3.5$ m

$z = 3$ m