MINISTRY OF HIGHER EDUCATION

THE HIGHER INSTITUTE FOR ENGINEERING & TECHNOLOGY IN NEW DAMIETTA

Department: Basic Science		SUMMER SEMESTER 2017-2018
COURSE TITEL: Math 1 COURSE CODE: MTH 101		DATE: 28 / 7 / 2018 DAY: SATERDAY TIME ALLOWED: 1½ hours
		No. of exam pages: 4 pages (1 sheet)
Total Mark: **20**	Marks:	Midterm Exam Model answer	اسم الطالب:
			كود الطالب:

Question 1 (10 marks) **Q1**

a) **Define** the domain and the range of each of the functions given in the following table (2 marks)

The Function	The Domain	The Range
$f(x) = \sqrt{3x-6}$	$[2, \infty)$	$[0, \infty)$
$f(x) = \tan^{-1}x$	$R - \{\frac{\pi}{2} + n\pi\}, n \in Z$	R
$f(x) = \ln(x-1)$	$(-1, \infty)$	R
$f(x) = \cos(x+5)$	R	$[-1,1]$

b) **Find** $f^{-1}(x)$, where $f(x) = \frac{2x-1}{3}$. (2 marks)

$$y = \frac{2x-1}{3}$$

$$x = \frac{3y+1}{2}$$

$$y = \frac{3x+1}{2}$$

$$f^{-1}(x) = \frac{3x+1}{2}$$

c) **Evaluate** the following limits: (3 marks)

1) $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right)$

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$$

$$-x^2 \leq \cos\left(\frac{1}{x}\right) \leq x^2$$

$$\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$$

Hence, using the squeeze theory we get: $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$

2) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} \frac{x^2+4x-12}{x^2-2x} & x \neq 2 \\ 6 & x = 2 \end{cases}$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^2+4x-12}{x^2-2x}$$

$$= \lim_{x\to 2} \frac{(x+6)(x-2)}{x(x-2)} = 4$$

3) $\lim_{z\to 1} \frac{6-3z+10z^3}{7z^3-2z^2+1}$

$$\lim_{z\to 1} \frac{6-3z+10z^3}{7z^3-2z^2+1} = \frac{6-3+10}{7-2+1} = \frac{31}{6}$$

d) **Discuss** the continuity of the function $f(x) = 2x^3 - 5x^2 - 10x + 5$ in the interval [-1 , 2]. Hence, **show** that $f(x)$ has a root in that interval. (3 marks)

The function $f(x)$ is a polynomial, then it is continuous for all real numbers (R).
Hence, the function $f(x) = 2x^3 - 5x^2 - 10x + 5$ is continuous in the interval [-1 , 2].

To show that $f(x)$ has a root in that interval,

$$f(-1) = -2 - 5 + 10 + 5 = 8$$

$$f(2) = 16 - 20 - 20 + 5 = -19$$

Where $0 \in [-19,8]$, then $f(x)$ has a root in that interval [-1 , 2].
According to the intermediate theory.

Question 2 (10 marks) Q2

a) **Determine**, if the function $f(t)=\begin{cases} 4-t^2 & t<1 \\ (t-1)^2 & 1<t<2 \\ 1 & t\geq 2 \end{cases}$, is continuous or discontinuous at the points: $t=1$, $t=1.5$ and $t=2$. (2 marks)

At $t=1$:

$f(t)$ is not defined at $t=1$

Then $f(t)$ is not continuous at $t=1$

At $t=1.5$:
at $t=1.5$ $f(t)=(t-1)^2$, which is a polynomial. Hence, $f(t)$ is continuous at $t=1.5$.

At $t=2$:
1) $f(2)=1$
2) $\lim_{t\to^+2} f(t)=1$ and $\lim_{t\to^-2} f(t)=1$
That is $\lim_{t\to 2} f(t)=1$
3) $f(2)=\lim_{t\to 2} f(t)$
Then $f(t)$ is continuous at $t=2$

b) **Find** $\frac{dy}{dx}$, for each of the following functions: (6 marks)

1) $y=\pi^x x^{3\pi}-\cosh(\ln(3x^2))+\sqrt{x+\sqrt{x}}$

$$\frac{dy}{dx}=[\pi^x\cdot 3\pi\cdot x^{3\pi-1}+\pi^x\cdot\ln\pi\cdot x^{3\pi}]-[\sinh(\ln(3x^2))\frac{6x}{3x^2}]+\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}$$

2) $y=(\ln x)^x$

$$\ln y = x\ln(\ln x)$$

$$\frac{1}{y}\acute{y}=\ln(\ln x)+x\frac{1/x}{\ln x}=\ln(\ln x)+\frac{1}{\ln x}$$

$$\acute{y}=(\ln x)^x[\ln(\ln x)+\frac{1}{\ln x}]$$

3) $xy = (x^2 + y^2)^{3/2}$

$$x\acute{y} + y = \frac{3}{2}(x^2 + y^2)^{1/2}(2x + 2y\acute{y})$$

$$\acute{y} = \frac{3(x^2 + y^2)^{1/2} - y}{x + 3y(x^2 + y^2)^{1/2}}$$

4) $x = a(1 + \sin\theta), \quad y = a(1 - \cos\theta)$, where a is any real number.

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a(\sin\theta)}{a(\cos\theta)} = \frac{x-a}{a-y}$$

c)The amount of air in some balloon at any time t, is given by the function: $V(t) = \frac{6\sqrt[3]{t}}{4t+1}$;

Determine if that amount is increasing or decreasing at t=8? (that is: if the balloon is being filled with air or being drained of air) (2 mark)

$$\acute{V}(t) = \frac{2(4t + 1)\, t^{-2/3} - 24\, t^{1/3}}{(4t + 1)^2}$$

at t=8

$$\acute{V}(8) = \frac{2(32 + 1)\, 8^{-2/3} - (24)\, 8^{1/3}}{(32 + 1)^2} = -0.029$$

Which has a negative sign

The amount of the air in the balloon is decreasing

That is the balloon is being filled drained of air

With my best wishes.

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